
Gravitational Fields

Examples

2002

Currently, the space probe, Cassini, is **between** Jupiter and Saturn. Cassini's mission is to deliver a probe to one of Saturn's moons, Titan, and then orbit Saturn collecting data. Below is astronomical data that you may find useful when answering the following questions.



mass of Cassini	$2.2 \times 10^3 \text{ kg}$	mass of Jupiter	$1.9 \times 10^{27} \text{ kg}$
mass of Saturn	$5.7 \times 10^{26} \text{ kg}$	Saturn day	10.7 hours

Question 3

Calculate the magnitude of the total gravitational field experienced by Cassini when it is $4.2 \times 10^{11} \text{ m}$ from Jupiter and $3.9 \times 10^{11} \text{ m}$ from Saturn.

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Solution Q3

Gravitational field strength is given by $g = \frac{GM}{R^2}$ - in this case we have a 'combined' field due to Saturn, to the left and Jupiter to the right.

$$\text{Jupiter } g = \frac{6.67 \times 10^{-11} \times 1.9 \times 10^{27}}{4.2 \times 10^{11}} = 7.18 \times 10^{-7} \text{ to the right.}$$

$$\text{Saturn } g = \frac{6.67 \times 10^{-11} \times 5.7 \times 10^{26}}{3.9 \times 10^{11}} = 2.50 \times 10^{-7} \text{ to the left}$$

Difference between the two is $4.68 \times 10^{-7} \text{ N kg}^{-1}$. You don't need to state the direction.

Examiner's comment

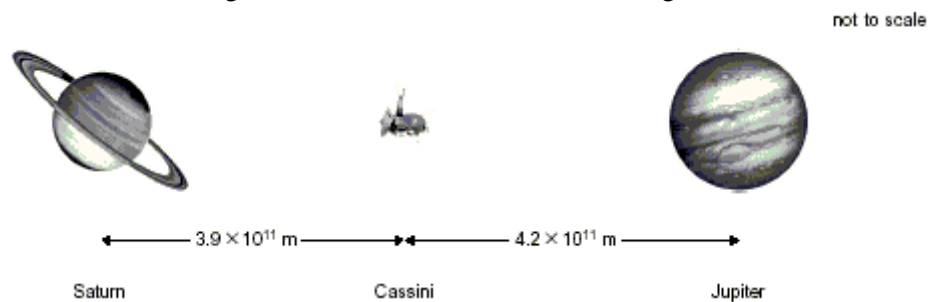
The expected answer for this question involved subtracting the gravitational field due to Saturn from that due to Jupiter according to the equation. This resulted in a value for the gravitational field strength of $4.7 \times 10^{-7} \text{ N kg}^{-1}$.

It was not expected that students would include the gravitational field of the sun even though this turns out to be significantly greater at about $2 \times 10^{-4} \text{ N kg}^{-1}$ at this point (students who did include the effect of the sun were fully rewarded).

The most common error was to calculate the field values and then add them rather than subtract. Another common error, becoming quite frequent in the past few years, is to neglect to square the radius value in the calculation. A number of students made arithmetic errors at some stage of the calculation.

Question 4

Indicate the direction of the gravitational field at Cassini on the figure below.



Cassini between Saturn and Jupiter (not drawn to scale)

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Solution Q4

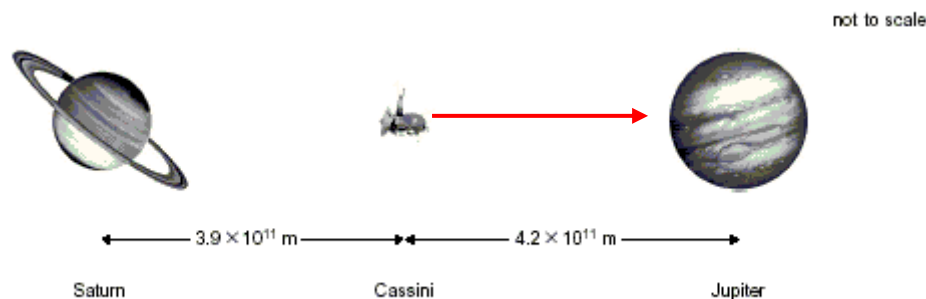
Gravitational field strength is given by $g = \frac{GM}{R^2}$ - in this case we have a 'combined' field due to Saturn, to the left and Jupiter to the right.

$$\text{Jupiter } g = \frac{6.67 \times 10^{-11} \times 1.9 \times 10^{27}}{4.2 \times 10^{11}} = 7.18 \times 10^{-7} \text{ to the right.}$$


$$\text{Saturn } g = \frac{6.67 \times 10^{-11} \times 5.7 \times 10^{26}}{3.9 \times 10^{11}} = 2.50 \times 10^{-7} \text{ to the left}$$

Difference between the two is $4.68 \times 10^{-7} \text{ Nkg}^{-1}$.

∴ The net result is towards Jupiter

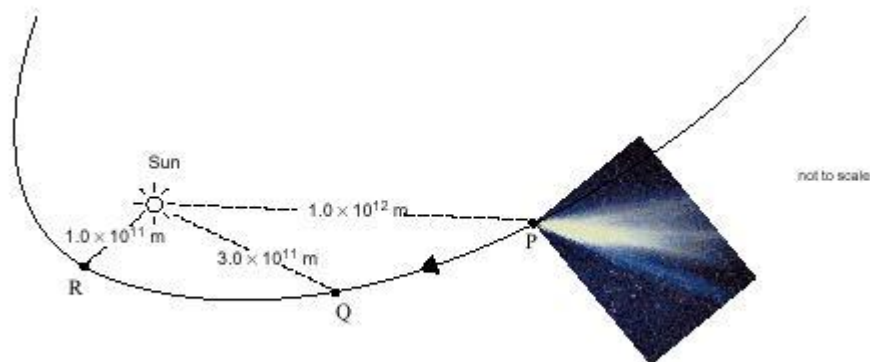


Examiner's comment

Students were expected to show an arrow in the direction  at the position Cassini. A common error was to draw two arrows, a small one to the left and a larger one to the right.

2000

Halley's comet last passed by Earth in 1986. The path of this comet is elliptical. Part of this ellipse is shown in Figure 1, with P, Q and R representing three points along the path. Point P is $1.0 \times 10^{12} \text{ m}$ from the Sun.



Question 1

Calculate the magnitude of the gravitational field strength at point P due to the Sun.

($G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$, $M_{\text{sun}} = 2.0 \times 10^{30} \text{ kg}$)

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Solution Q1

g is the gravitational field strength. So using; $F = mg = \frac{GMm}{R^2}$, the m cancels and we can write

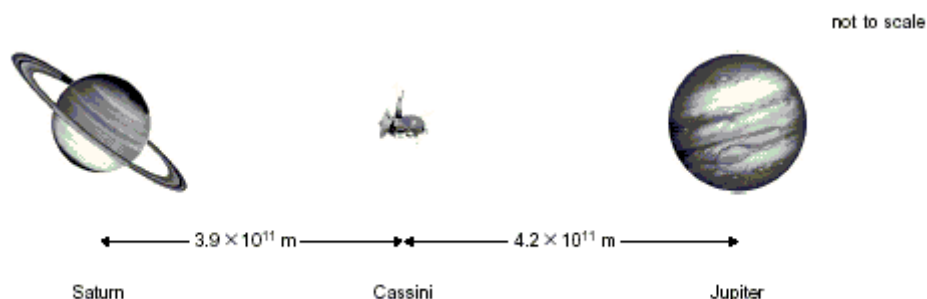
$$g = \frac{GM}{R^2} \quad \text{Sub the values in} \quad g = \frac{6.67 \times 10^{-11} \times 2.0 \times 10^{30}}{(1.0 \times 10^{12})^2} \quad \therefore g = 1.33 \times 10^{-4} \text{ N kg}^{-1}$$

Examiner's comments

The magnitude of the sun's gravitational field at point P calculates to $1.33 \times 10^{-4} \text{ N kg}^{-1}$. This required a simple substitution into the field strength equation. With an average score of 1.59/2, most students understood how to calculate the gravitational field strength. The most common error occurred as a result of students neglecting to square the distance; that is, they correctly wrote down the formula and substituted in values, but then forgot to square. It is also worth encouraging students to consider the likeliness of answers (forgetting to square the distance obtained an answer that was too large).

Circular Orbits

Examples 2002



mass of Cassini	$2.2 \times 10^3 \text{ kg}$	mass of Jupiter	$1.9 \times 10^{27} \text{ kg}$
mass of Saturn	$5.7 \times 10^{26} \text{ kg}$	Saturn day	10.7 hours

When Cassini arrives in the vicinity of Saturn this year, scientists want it to remain above the same point on Saturn's equator throughout one complete Saturn day. This is called a 'stationary' orbit.

Question 5

What is the period in seconds of this 'stationary' orbit?

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Solution Q5

Satellite needs to have same period as a day on Saturn. ie 10.7 days.

In seconds this equals $3.85 \times 10^4 \text{ s}$.

Examiner's comment

To remain above the same point on Saturn's equator the satellite would be required to have a period of 10.7 hours, or $3.85 \times 10^4 \text{ s}$. The main difficulties were to either assume a 24-hour day or to make an arithmetic error in the calculation.

Question 6

Calculate the radius of this 'stationary' orbit. ($G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$)

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Solution Q6

You need to use the variation of the usual equation $\frac{GMm}{r^2} = \frac{m4\pi^2 r}{T^2}$. In this case, the mass m of Cassini

cancels out. So the equation becomes $\frac{GM}{r^2} = \frac{4\pi^2 r}{T^2}$.

$$\begin{aligned} \text{On substitution this becomes } \frac{6.67 \times 10^{-11} \times 5.7 \times 10^{26}}{r^2} &= \frac{4\pi^2 \times r}{(3.85 \times 10^4)^2} \\ \therefore r^3 &= \frac{6.67 \times 10^{-11} \times 5.7 \times 10^{26} \times (3.85 \times 10^4)^2}{4\pi^2} \\ \therefore r^3 &= 1.442 \times 10^{24} \\ \therefore r &= 1.13 \times 10^8 \text{ m} \end{aligned}$$

Examiner's comment

Application of Newton's Law of Universal Gravitation for the force between two masses along with the relation for uniform circular motion resulted in the equation: $\frac{GMm}{r^2} = \frac{m4\pi^2r}{T^2}$

Substitution of the appropriate values resulted in a radius of 1.1×10^8 m for the stationary orbit. Many students experienced difficulty with the concept of a stationary orbit. Others had difficulty getting started, often starting with Newton's Law of Universal Gravitation but were unable to combine this with the circular motion equation involving the period. Students are more comfortable with the relation

$$\frac{mv^2}{r} \text{ but not so familiar with } \frac{m4\pi^2r}{T^2}$$

For those who could successfully write down and substitute into the formula, many made arithmetic errors. The final stage of taking the cube root to find **R** was very poorly done.

2001



The Mir space station orbited Earth at an **altitude** of 390 km. The total mass of the space station was 140 tonnes and Mir completed 83 500 orbits in the 14.6 years before it crashed to Earth on 23 March 2001.

Question 1

Calculate the period of Mir's orbit in seconds.

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Solution Q1

If the space station completed 83 500 orbits in 14.6 years, then the period is given by

$$\frac{\text{time(secs)}}{\text{orbits}} = \frac{14.6 \times 365 \times 24 \times 60 \times 60}{83500} = 5.514 \times 10^3 \text{ secs} \quad \therefore 5.5 \times 10^3 \text{ s}$$

Examiner's comment

Question 1 (1.65/3)

The period of Mir's orbit was $5.5 \times 10^3 \text{ s}$.

This was a relatively simple question and the most common problems were either due to simple arithmetic mistakes or to substituting of incorrect numerical values when converting 14.6 years into seconds. Some students took a 'round-about' approach and went back to the equations for Universal Gravitation and circular motion, often making arithmetic errors when doing so.

Question 2

Calculate the speed of Mir while in orbit.

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Solution Q2

You need to be careful with the radius of orbit. The station is 390 km above the surface of the Earth, so the radius of orbit is $R_E + 390$ km. The units are mixed, so you need to be careful. $R_{\text{orbit}} = 6.37 \times 10^6$ m

$$v = \frac{2\pi r}{t} = \frac{2\pi(6.37 \times 10^6 + 390 \times 10^3)}{5.5 \times 10^3} = 7703 \text{ ms}^{-1} = 7.7 \times 10^3 \text{ ms}^{-1}$$

Examiner's comment

Question 2 (1.65/3)

The speed of Mir while in orbit was $7.7 \times 10^3 \text{ m s}^{-1}$.

This question was reasonably well answered, particularly by those students who worked with the given data rather than trying to work with equations from the data booklet or their A4 sheet. Common errors resulted from failing to include the altitude when determining the radius of the orbit and the usual problem of calculating arithmetic expressions involving powers of 10.

Question 3

Calculate the value of the gravitational field strength at 390 km above Earth's surface.

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Solution Q3

Since it is in circular motion, $a = g = \frac{v^2}{r}$ where g is the gravitational field strength.

$$\frac{(7.7 \times 10^3)^2}{6.37 \times 10^6 + 390 \times 10^3} \quad g = \frac{59.29 \times 10^6}{6.76 \times 10^6} = 8.78 \text{ N kg}^{-1}$$

Examiner's comment

Question 3 (2.6/5)

For uniform circular motion the acceleration can be calculated via application of the equation $a = v^2/r$, using the speed calculated in Question 2 and the radius of the orbit as the radius of the earth plus the altitude.

This resulted in a gravitational field value of 8.8 N kg^{-1} .

The question itself was reasonably well understood by most students. The two most common errors were in failing to include the altitude when determining the orbit radius and using the mass of Mir rather than Earth for the calculation. Students need to be aware that it is the central mass, not that of the orbiting body, that determines gravitational field strength and they need to be reminded to think more carefully when choosing values to substitute into formulas.

When Mir was in stable orbit, an object of mass 2.5 kg was placed on a spring balance that was attached to the inside of the space station.

Question 4

What would be the weight shown on the spring balance? Explain your reasoning, or show your working, for determining the reading of the spring balance.

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Solution Q4

Since Mir is in a stable circular orbit, and $a = g$, then the reaction force from the spring balance should be zero. The net force = $mg - T$.

$$\text{Or } mg - T = ma$$

This must equal ma . But $a = g$, so $T = 0$.

$$\therefore mg = ma = T \quad \therefore T = 0.$$

Examiner's comment

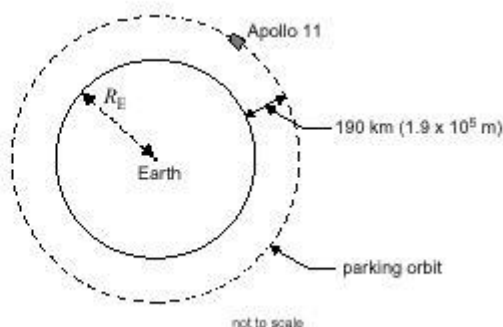
Question 4 (2.6/5)

This question was testing the concept of apparent weightlessness in a slightly different format than in previous examinations. Both the spring and the mass are accelerating at the same rate and hence the reading of the spring balance will be zero.

This proved to a reasonably demanding question and certainly highlighted just how poorly students understand the concept of apparent weightlessness. The most common error was to give the answer as the weight force mg (22 N). Very few students mentioned that the net force on the mass was mg and hence the spring force was zero. Very few chose to draw a force diagram and use this to calculate the spring force as zero. Many answers referred to 'freefall' or 'apparent weightlessness' but without using this to really address the question asked. Many students inappropriately used the term 'normal reaction' rather than 'tension' when referring to the force exerted by the spring.

2000

When people went to the Moon in the Apollo 11, the spacecraft was initially placed in a 'parking orbit' 190 km above Earth's surface. This is shown below.



Question 3

Calculate the speed of Apollo 11 in the parking orbit.

$$(R_E = 6.37 \times 10^6 \text{ m}, M_E = 5.98 \times 10^{24} \text{ kg}, G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})$$

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Solution Q3

In order to do this question you need to understand that the net force on the spacecraft is the gravitational force, and that it is this gravitational force is the centripetal force required to keep the spacecraft in orbit. Therefore you can write;

$$F_{\text{net}} = \frac{GMm}{R^2} = m \frac{v^2}{R} \quad \text{divide both sides by } m$$

$$\frac{GM}{R^2} = \frac{v^2}{R} \quad \text{times both sides by } R$$

$$\frac{GM}{R} = v^2 \quad \text{take the square root of both sides}$$

Before you sub the values in make sure they are the right ones

Lets establish what R is; it is the distance from the centre of mass of the Earth to the centre of mass of the spacecraft. So it's the radius of the earth plus the distance the spacecraft is above the Earth's surface.

$$R = R_E + 1.9 \times 10^5 \quad R = 6.37 \times 10^6 + 1.9 \times 10^5 \quad \therefore R = 6.56 \times 10^6 \text{ m}$$

M is the mass of the earth = $5.98 \times 10^{24} \text{ kg}$

Now you can sub the values in and you get

$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{6.56 \times 10^6}} \quad \therefore v = 7.8 \times 10^3 \text{ ms}^{-1}$$

Examiners comments Q3

The parking orbit of Apollo 11 is an example of uniform circular motion in which the net force is provided by the gravitational force between Earth and Apollo 11. This resulted in a speed of $7.8 \times 10^3 \text{ ms}^{-1}$.

The average mark of 1.76/3 demonstrates that just over 50% students had a reasonable grasp of this concept. In fact, about half of the students scored the full 3 marks for this question, but about 30% scored zero. The most common error was made by students forgetting to add the spacecraft's altitude to the radius of Earth when determining the radius of the orbit. Many students made simple errors when substituting in numerical values or when using their calculator.

Two students were discussing the physics of the motion of Apollo 11 in the parking orbit. They realised that it was travelling at a constant speed but **not** in a straight line. Jane said that this can be predicted by Newton's first law. Maria claimed that this disobeyed Newton's first law.

Question 4

Explain, giving reasons, whether Jane is correct or incorrect, **and** whether Maria is correct or incorrect.

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Solution Q4

- A body travelling in a straight line will continue to travel in a straight line unless a net force acts on it.
 - the velocity of Apollo 11 was changing (change in direction), \therefore it was accelerating
 - if Apollo 11 was accelerating there must have been a net force acting on Apollo 11.
- \therefore Jane was correct, and Newton's first Law was not broken. You must answer the question by making statement about Jane and/or Maria

Examiners comment Q4

In order to address whether Jane or Maria was correct students were expected to discuss the following points:

- a statement of Newton's First Law
- the velocity of Apollo 11 was changing (change in direction), hence it was accelerating
- a net force (gravitational force) was acting on Apollo 11.

The average score here was 1.16/3. It is clear that students are able to recall Newton's Laws but were unable to apply them in a particular physical situation. Few students discussed the net force acting on Apollo 11 or the fact that it was accelerating.

Question 5

Which **one** of the following statements (A–D) best describes the origin of the centripetal force required to keep Apollo 11 in its circular parking orbit?

- A. Apollo 11's acceleration towards Earth
- B. the rocket motors of Apollo 11
- C. the speed of Apollo 11 in its orbit
- D. the gravitational field of Earth acting on Apollo 11

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Solution Q5

This question is asking where is the centripetal force required to keep the spacecraft in orbit coming from. The only statement that answers this question is D, the gravitational field of the Earth acting on Apollo 11.

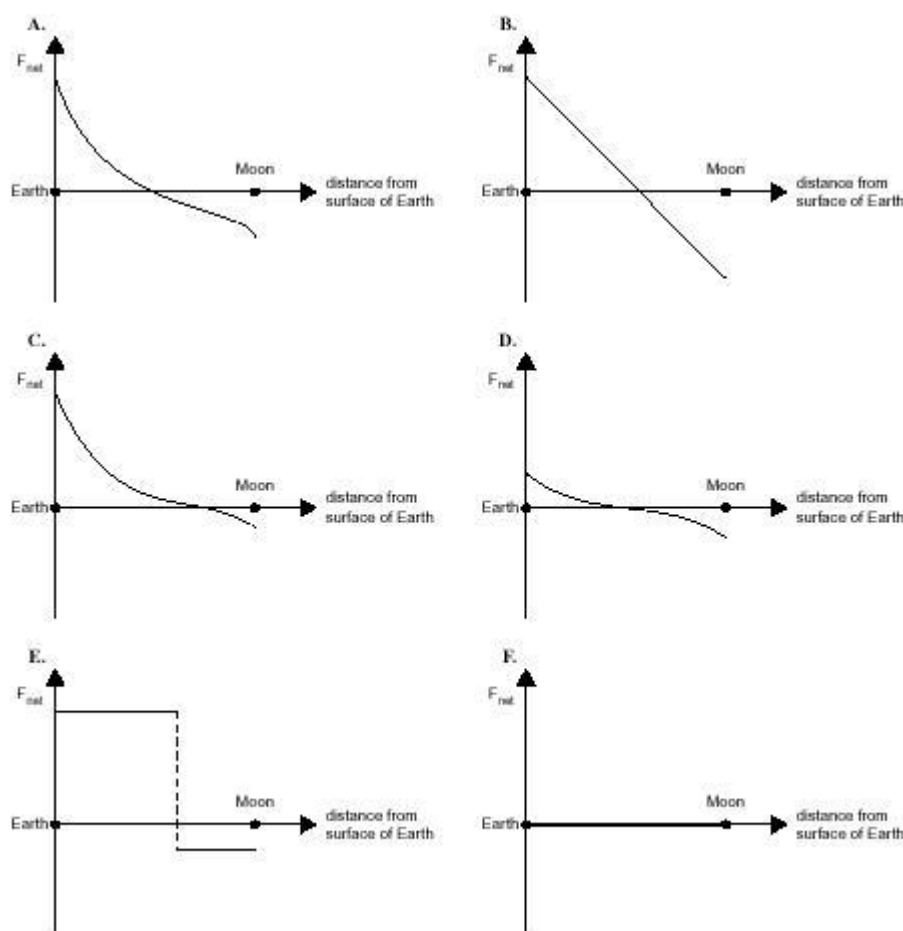
Examiners comment Q5

The origin of the net force needed to support uniform circular motion of Apollo is that of the gravitational attraction exerted by Earth on Apollo 11. Hence, statement D was correct. The average mark of 0.57/1 was a disappointing result for such a simple question.

Apollo 11 then leaves its parking orbit and travels to the Moon.

Question 6

Which **one** of the following graphs (A–F) best represents the **net gravitational force** acting on Apollo 11 as it travels from its parking orbit to the Moon?



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Solution Q6

Graph C best represents the net force acting on Apollo 11 as it travels from the parking orbit to the moon. The things to know would be the force is not linear, so B, E and F are out, when the force from the Earth equals the force from the Moon, the spacecraft will be closer to the moon then the Earth, and it will not occur half way between the two objects. This rules out A and D therefore C is the most reasonable answer.

Examiners comment Q6

Graph C best represents the variation of the net gravitational force acting on Apollo 11. The force decreases according to the inverse square law as Apollo 11 travels away from Earth, reaching zero at a point closer to the Moon. The direction of the force now changes as Apollo 11 experiences a net force directed towards the moon. The magnitude of this net force again increases as Apollo 11 approaches the Moon.

The average mark here was 1.17/2, which was reasonable for what was anticipated to be a more difficult question.

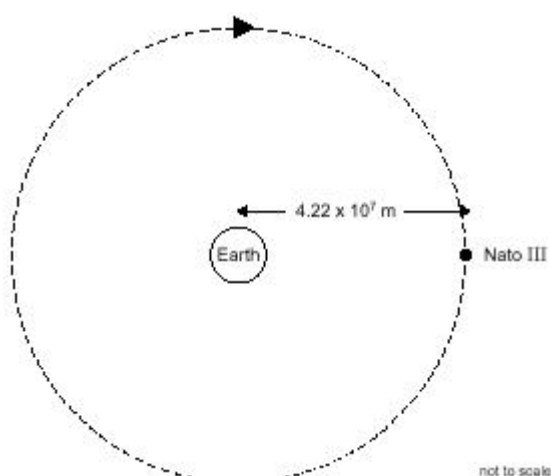
1999

Nato III is a communication satellite that has a mass of 310 kg and orbits Earth at a constant speed at a radius of 4.22×10^7 m from the centre of Earth.

Question 1

Calculate the magnitude of Earth's gravitational field at the orbit radius of Nato III.

Give your answer to **three significant figures**. You **must** show your working.



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Solution Q1

The gravitational field strength is given $g = \frac{GM_e}{R^2}$ by

$$\text{Substituting gives } g = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(4.22 \times 10^7)^2} = 0.223976 \quad \therefore$$

$$g = 0.224 \text{ ms}^{-2}$$

Examiners comment Q1

The magnitude of Earth's gravitational field calculates out to 0.224 N kg^{-1} . This required a simple substitution into the field strength equation and with an average score of 1.5/2 a majority of the students were able to do this. The vast majority of students recognised the correct formula to use and any errors were due to using the incorrect value for the mass or being unable to correctly manipulate the data when using the calculator. A number of candidates failed to answer to three significant figures as requested in the question.

Question 2

What is the speed of Nato III in its orbit?

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Solution Q2

The gravitational field strength equals the acceleration of the satellite. Thus

$$\frac{v^2}{R} = 0.224 \Rightarrow v = 3.07 \times 10^3 \text{ ms}^{-1}$$

Examiners comment Q2

The speed of Nato III calculates to $3.1 \times 10^3 \text{ m s}^{-1}$. This could have been achieved by simply combining the formulas of Universal Gravitation and uniform circular motion. The average mark for this question was 1.9/3 indicating that the majority of candidates understood how to derive the necessary equation and then correctly calculate the speed. The equation for the speed can be derived directly and many students did so. However, it was disappointing to note the number of students who started by calculating the period of the orbit and then determining the speed in what was a two-step process. This latter method not only wasted valuable exam time but increased the likelihood of an arithmetic error as well.

This question also highlighted the continuing problem that many students have when using the calculator to determine the value of expressions with indices.

Question 3

Which **one** of the following statements (A–D) about Nato III is correct?

- A. The net force acting on Nato III is zero and therefore it does not accelerate.
- B. The speed is constant and therefore the net force acting on Nato III is zero.
- C. There is a net force acting on Nato III and therefore it is accelerating.
- D. There is a net force acting on Nato III, but it has zero acceleration.

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Solution Q3

C

If there was no net force on the satellite it would travel in a straight line rather than orbit.

Examiners comment Q3

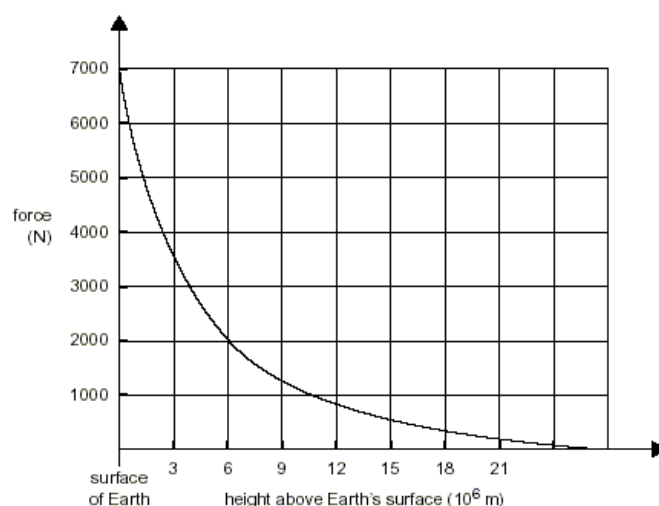
Nato III is travelling in uniform circular motion and hence there is a net force, provided by the gravitational attraction to Earth, acting towards the centre of the orbit (Earth). Thus, statement C was correct. The average mark of 1.0/2 demonstrates t

Gravitational Energy Transfers

Examples

2002

The Mars Odyssey spacecraft was launched from Earth on 7 April 2001 and arrived at Mars on 23 October 2001. Figure 1 is a graph of the gravitational force acting on the 700 kg Mars Odyssey spacecraft plotted against height above Earth's surface.



Question 1

Estimate the minimum launch energy needed for Mars Odyssey to escape Earth's gravitational attraction.

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Solution Q1

Area under graph is something like 11 to 13 squares

Each square has a value of $1000 \times 3 \times 10^6 = 3 \times 10^9$ J \Rightarrow at least 3.3×10^{10} J

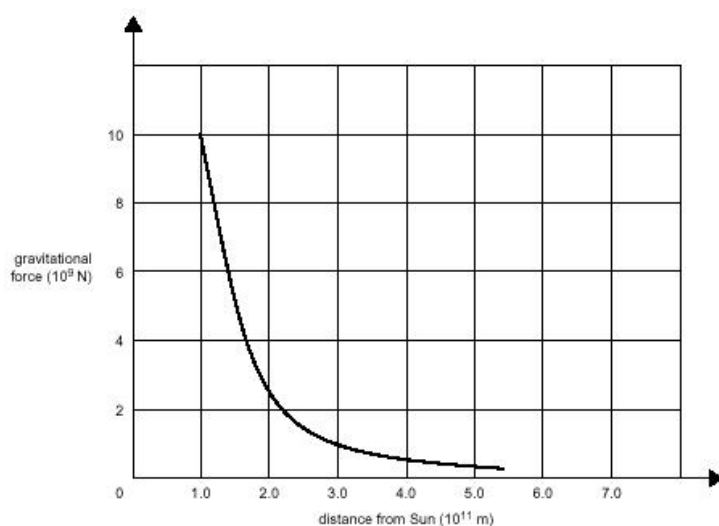
Examiner's comments

The required launch energy was calculated by determining the total area under the graph. Square counting resulted in approximately 13 squares, with each square representing a work done of 3×10^9 J. Hence, the total energy required was $13 \times 3 \times 10^9 = 3.9 \times 10^{10}$ J. Allowing for a variation in the number of squares counted, a range of values 3.3 to 4.4×10^{10} J, was accepted.

Most students recognised that the area under the graph was the key to answering this question. The most common error was incorrectly calculating the area of each square on the graph, usually by neglecting the 10^6 for the height axis. Others made an error in their estimation of the number of squares, usually in counting too few squares. Area estimation may need reviewing for some students. Some students lost a mark due to multiplying their calculated area by 700 kg, obviously being confused between force and field. It should be noted that the study design specifically mentions that it is force-distance graphs only that are to be examined in this context.

2000

Figure 2 shows a graph of the gravitational force that the Sun exerts on Halley's comet as a function of the distance of the comet from the Sun.



2000 Question 2

Estimate the increase in kinetic energy of the comet as it travels from Q (3.0×10^{11} m from the Sun), to R (1.0×10^{11} m from the Sun). Show your working.

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Solution Q2

To do this question you must estimate the area under the force distance graph. I did this by counting the squares under the graph between the points 3.0×10^{11} m and 1.0×10^{11} m. Each square represents 2.0×10^{20} J. I counted roughly $3\frac{3}{4}$ squares. Which means there is roughly 7.5×10^{20} J.

Examiners comment Q2

The increase in kinetic energy was determined from the area under the force-distance graph between the distance values of 1.0×10^{11} m and 3.0×10^{11} m. This could be achieved by estimating the area by square counting or by a suitable geometric approximation. This resulted in an increase of 7.5×10^{20} J. (values between 7.0×10^{20} J and 8.0×10^{20} J were accepted as reasonable approximations).

Weight and Apparent Weight

Examples

2001

On 12 February 2001, a spacecraft named the NEAR Shoemaker landed on Eros, a peanut-shaped asteroid (pictured) between the orbits of Earth and Mars. The 450 kg spacecraft had a weight of 2.5 N when it landed on the asteroid.



Eros

Question 5

Calculate the weight of this spacecraft on the surface of Earth.

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Solution Q5

$$W = mg = 450 \times 9.8 = 4410\text{N} = 4.4 \times 10^3 \text{ N}$$

Examiner's comments

Question 5 (1.68/2)

The weight of the spacecraft on the surface of Earth was $4.4 \times 10^3 \text{ N}$.

This question was quite well understood by most students. A small number of students misread the question and calculated g for the surface of Eros.

Before landing on Eros, the spacecraft orbited at a radius of 50 km from the centre of mass of Eros, with a period of $5.9 \times 10^4 \text{ s}$ (about 16 hours).

Question 6

Find the mass of Eros from this information.

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Solution Q6

The satellite was in a stable circular orbit.

$$\begin{aligned} \therefore \frac{GMm}{r^2} &= \frac{mv^2}{r} & \text{Since } v = 2\pi r/T, \text{ then } \frac{mv^2}{r} \text{ becomes } \frac{4\pi m r^2}{r T^2} &= \frac{4\pi m r}{T^2} \\ \therefore \frac{GMm}{r^2} &= \frac{4\pi m r}{T^2} & \therefore M &= \frac{4\pi r^3}{GT^2} = \frac{4\pi(50 \times 10^3)^3}{6.67 \times 10^{-11} \times (5.9 \times 10^4)^2} = 6.7 \times 10^{15} \text{ kg} \end{aligned}$$

Examiner's comments

The average score for Questions 6 and 7 was a combined 2.1/4 indicating this was a moderately demanding group of questions (30% scored the full 4 marks).

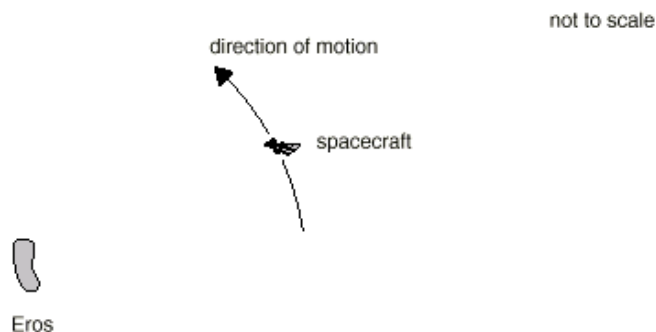
Question 6

Application of the equations for universal gravitation and uniform circular motion resulted in a mass for Eros of 2.1×10^{16} kg.

This question was not well done and the errors were many and varied, with a number of incorrect formulas. Students also made some simple errors, such as forgetting to convert 50 km into metres or using the value of g for the surface of Eros rather than at the radius of the orbit. Many students experienced difficulty using powers of 10 in their calculations and/or forgot to cube or square the numbers.

Question 7

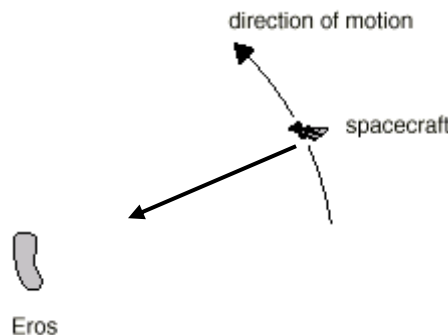
On the diagram below, draw one or more labelled arrows to show any force(s) acting on the spacecraft as it orbits Eros. You can ignore any other astronomical bodies.



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Solution Q7

Since the spacecraft was in a stable orbit, the net force must be acting radially inwards. This is the only force acting.



Examiner's report

Question 7

There is only one force acting, that of the gravitational force that Eros exerts on the spacecraft. Hence, an arrow pointing inwards from the spacecraft towards Eros was expected.

This was reasonably well done with most common errors being:

- drawing two arrows, one toward and one away from Eros
- drawing an arrow vertically down to represent the weight force and drawing a tangential arrow and then omitting to label this as representing the direction of motion.

1999

The Russian space station, MIR (Russian meaning, *peace*), is in a circular orbit around Earth at a height where the gravitational field strength is 8.7 N kg^{-1} .

Question 4

Calculate the magnitude of the gravitational force exerted by Earth on an astronaut of mass 68 kg on board MIR.

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Solution Q4

592 N

Force = mg so the force is $68 \times 8.7 = 592 \text{ N}$ [$5.9 \times 10^2 \text{ N}$] in other words, the astronaut's weight!

Make sure that you use the value for g (given as 8.7 N kg^{-1} , **not** 9.8 N kg^{-1})

Examiners comment Q4

Application of the formula $F = mg$ resulted in a gravitational force of 592 N. The average score here was 1.4/2, but it was disappointing to note that 30% of candidates scored zero for this question. It was of concern to note the number of students who tried to solve this question by using the law of Universal Gravitation rather than the more direct method. Another common error was to take g as 9.8 N kg^{-1} rather than the given value of 8.7 N kg^{-1} .

When the astronaut wishes to rest he has to lie down and strap himself into bed.

Question 5

What is the magnitude of the force that the bed exerts on the astronaut before he begins to fasten the strap?

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Solutions Q5

0 N

Force will be zero since MIR and the astronaut are both in state of free fall. The astronaut and the space station are both accelerating at the same rate, hence there is no contact force between them. The bed does not exert a force on the astronaut.

Examiners comment Q5

The astronaut and the space station are both accelerating at the same rate and hence there is no contact force between them. Hence, the bed does not exert a force on the astronaut. The average mark of 0.9/2 indicates that about only 45% of candidates correctly understood this concept.

Newspaper articles about astronauts in orbit sometimes speak about zero gravity when describing weightlessness.

Question 6

Explain why the astronaut in the orbiting MIR is not really weightless.

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Solution Q6

The astronaut is not weightless because the Earth is still attracting him/her. The astronaut still has weight, since $W = mg$ is the gravitational force applied to the astronaut by the Earth. In this situation g is not zero.

The weight force is merely providing the required force to travel in the circular orbit. The lack of reaction force from his/her environment results in an 'apparent weight' of zero.

Examiners comment Q6

In order to explain why the astronaut is not really weightless students needed to address these points:

- The astronaut actually has weight, since $W = mg$.
- In this situation, g is not zero
- Weightlessness refers to apparent weight, which is the value of the contact force discussed in the previous question.

The average mark here was 1.5/3, and the main problem noted was the number of students who devoted their answer to a discussion of “weightlessness” rather than simply addressing why the astronaut does have weight.